Problem **13**

This problem, there's three scenarios. And we are asked to determine the angular velocity. Given that all these three problems have the same angular momentum about point O, and this angular momentum is 0.1 kilogram meter squared per second. So we're going to start with each case and work our way through case one, two, and three, and determine the angular velocity omega one, omega two and omega three respectively. Let's start with our coordinate system. X points to the right, Y points upwards, and a positive rotation is defined as counterclockwise. So let's start with scenario one. In this case, we have a disk that has a radius of 45 centimeters and a mass of 600 grams. It is rotating with an omega that is clockwise, so it's going to be in the negative k hat direction. And we're given that we have our angular momentum about O being equal to 0.1 kilogram meter squared per second. So, for case one, we define the angular momentum as H naught comma one. So this is H O. So this is h about O with the casein case number one. So this is going to be equal to I naught and omega one. Right. So, this I knot, sorry, this shouldn't be one.I know, one is I about O, in the first case, and omega one is the angular velocity of the first case. So again, all these letters are these numbers here, this is essentially denoting case one. So, in this case, this is going to be equal to 0.1 kilograms meters squared per second. Now, it's important to note that I've made this into a scalar equation, but the real equation would be that h vector h is equal to i omega. So, the unit vector of omega is going to be the same unit vector as h and this will come in later when we define omega as a unit vector. So, we are given this, we need to determine omega. So since we're given the right side of the equation, and we need to find omega, we need to define this i naught comma one IO comma one. So, what is IO comma one that is the inertia about this point O at the bottom here. So, for a thin disk, we know that this is going to be equal to one half m r squared plus m one r squared. So, this is essentially using parallel axis we know that a rotating disc about its center is one half m r squared. And then if we move a distance of one radius downwards, so, m L squared where L is r, we get the inertia about point O. So, note again, this m one is essentially the mass in the first case, so that would be the 600 grams. So, we plug this in and we isolate for omega and we get the following equation. Omega one is equal to 0.1 kilograms, meters squared per second, divided by one half times 0.6 kilograms times 0.45 meters squared plus 0.6 kilograms times 0.45 meters squared. And we can finalize our answer to omega one being equal to 0.55 radians per second. So, to get a vectorial form, and look at the diagram, omega one, the vector of omega one is rotating in the clockwise direction, which means it's in the negative k hat direction, negative 0.55 radians per second in the k hat direction. This is our final answer for omega one.Okay, now we're gonna move on to the second case. So in the second case, instead of having a disc , rotating, we have a bar that's rotating, and this is pin at one end. And it's rotating with omega two, center of gravity is located halfway down the bar. So for case two, we have that H naught two is equal to I zero, I naught to omega two is going to be equal to 0.1 kilograms meters squared per second. Same thing as before. And we can find I knot too. So I knot two, that's going to be equal to. So we know that this is a bar that is pinned about the end. So we need to find the moment of inertia about this point. And that is just 1/3 m L squared. So that's going to be equal to 1/3 m two L squared, where this L over here is big L, right? The length of the beam, and that is 90 centimeters. So we plug everything in. And we get that omega two is equal to 0.1 kilograms, meters squared per second, divided by 1/3 0.35 kilograms, times 0.9 meters squared. And we get that omega two is equal to 1.06 radians per second. And the vectorial form omega two is equal to, if we look at this diagram, this is turning again in the clockwise direction. So this will be a, again, negative k hat direction. Negative 1.06 radians per second in the k hat direction. And this is our second case. So the answer for our second case. Lastly, let's look at the last case.

So the last case, it's again, a bar, that's at a distance d. But with its pin that O which is a point that is a quarter of the length away from the center of gravity, right. And this, we have omega three in this case, as the angular velocity. So we need to again, look at case number three. And H O three is equal to i o three, Omega three, this is equal again to 0.1 kilograms, meters squared per second, because we always have the same angular momentum right. Now, let's find i 03. And this is, again, for a bar. So for the bar, we have 1/12 m L squared, plus we use parallel axis to move by a distance of d over four to get to this point over here. So we have it being 1 12 m three, L squared, in this case, L is D plus parallel axis, M three times D over four, because we're just moving by a quarter, and then that squared. So if we plug this in, we get that omega three is equal to 0.1 kilograms, meters squared per second, divided by 112 times 0.63 kilograms, times 0.67 meters, all squared plus 0.63 kilograms times 0.67 divided by four all squared, close bracket. And once we plug in these values into our calculator, we see that omega three is equal to 2.42 radians per second. And if we look at the vectorial equation, omega three is equal to, in this case, we are rotating counterclockwise. So this is going to be a positive k hat direction, this case is going to be 2.42 radians per second in the k hat direction. And this is the final answer for our last scenario. Now comparing these, we can see that comparing specifically these two cases, if you have the same angular momentum, but you move this pin closer towards the center, you will end up with a larger angular velocity. And this direction doesn't really matter, positive or negative, because that's how we just set it up in the question with just arrows. But because in this question, we're assuming that the magnitude of the angular momentum is constant and not the direction, but we can see that the closer we move this pin to G, so the center of gravity, the larger the angular velocity, because all we're changing in this problem is i right inertia. So as you move closer, this inertia will get smaller. So then, your angular velocity will get larger to compensate and maintain this angular momentum constant